What do you means Statements/Proposition:-

Meaningful sentence either universal true or universal false is called statements/Propositions

Example:-

1:-Gravitational value is 9.8 is a true statement

2:-How wonderful!

3:-What did you say?

4:-34<3 is a false statement.

5:-6*4=10 is a false statement

Following symbols are used in propositions for statement operations.

Logical connectives:-

- **Disjunction** ∨
- **Conjunction** Λ
- **Negation** ¬ or ~
- **Conditional Connectives** →
- **Biconditional Connectives** ↔
- **Symetric Sum** ⊕

Example of Disjunction ∨

1:- Make a truth table of following connectives

P ∨ Q

P	Q	P∨Q
F	F	F
F	T	T
T	F	T
T	T	T

Example of Conjunction Λ

2:- Make a truth table of following connectives

P ^ Q

P	Q	P Λ Q
F	F	F
F	T	F
T	F	F
T	T	T

Example of Negation ¬

3:- Make a truth table of following connectives

¬ (P ^ Q)

P	Q	P^Q	¬ (P Λ Q)
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	F

Example of Conditional Connectives →

4:- Make a truth table of following connectives

(P → Q)

P	Q	(P → Q)
F	F	T
F	T	T
T	F	F
T	T	T

Example of BiConditional Connectives ↔

5:- Make a truth table of following connectives

(P ↔ Q)

P	Q	(P ↔ Q)
F	F	T
F	T	F
T	F	F
T	T	T

Example of Symmetric Connectives

6:- Make a truth table of following connectives

(P ⊕ Q)

P	Q	(P ⊕ Q)
F	F	F
F	T	T
T	F	T
T	T	F

Tautology:-

A compound proposition that is true for all possible truth values of the simple propositions involved in it is called a tautology.

Example:- F=P→ Q↔~P∨ Q

P	Q	~P	~P∨ Q	P→ Q	(P→ Q)↔(~P∨ Q)
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	F	T
T	T	F	T	T	T

Contradiction:-

A proposition that is false for all possible truth values of the simple propositions that constitute it is called a contradiction.

Example:-F=~ [P→ Q↔~Pv Q]

P	Q	~P	~P∨ Q	P→ Q	(P→ Q)↔(~P∨ Q)	~[(P→ Q)↔(~P∨ Q)]
F	F	T	T	T	T	F
F	T	T	T	T	T	F
T	F	F	F	F	T	F
T	T	F	T	T	T	F

Theorem based on Propositions/Statements:-

Demorgan's Law:-

~[A Λ B]=~A ∨ ~B

~[A ∨ B]=~A Λ ~B

Double Negation Law:-

~[~A]=A

Idempotent Law:-

A Λ A Λ A Λ A Λ A Λ A … Λ A = A
A ∨ A ∨ A ∨ A ∨ A ∨ A ….. ∨ A = A

Commutative Law:-

A Λ B = B Λ A
A ∨ B = B ∨ A

Associative Law:-

(A Λ B)ΛC=A Λ(B ΛC)
(A v B) v C=A v (B v C)

Distributative Law:-

A Λ (B v C) = (A Λ B) v (A Λ C)
A v (B Λ C) = (A v B) Λ (A v C)

Logical Quantifiers:-
A propositional function or predicate in a variable x is a sentence P(x) involving x that becomes a propositions when we give x a definite value from the set of values it can take.
Example:-

E(x) : All Employees working in Microsoft.
A(y) : Some employees are analyst in Microsoft.
D (z) : Some employees are database designer & Analyst in Microsoft.
Conclusions:-

Precedence Rule:-

First Priority	~
Second Priority	$\wedge$
Third Priority	$\vee$ and $\oplus$
Fourth Priority	$\rightarrow$ and $\leftrightarrow$

Methods of Proofs:-

Argument:-

It is a finite sequence of statements P_1, P_2, P_3 P_4 P_5... Pn Such that

$$P_1 \wedge P_2 \wedge P_3 \wedge P_4 \ldots \wedge P_n \rightarrow P$$

Where :-

P_1, P_2, P_3 P_4 P_5... Pn are called premises or assumption or Hypothesis.

And

The final statement P is called the conclusion.

Methods of Proofs:-

- Direct Method
- Indirect Method
- Counter examples
- Mathematical Induction

Direct Method:-

It is logically valid argument that begins with the assumptions that p is true and, in one or more applications of the law of detachment, conclude that must be true.

So, to construct a direct proof of $p \Rightarrow q$, we start by assuming that p is true.Then, in one or more steps of the form $p \Rightarrow q1$, $q1 \Rightarrow q2$, $q2 \Rightarrow q3$…, $qn \Rightarrow q$,
We concude that q is true.

Example:-Prove that product of two odd numbers is odd.
Solution:-

Let p=(2m+1)
q=(2n+1)

Where m and n are integer.

Solution:-

p*q = (2m+1)* (2n+1)
= (4mn +2m +2n+1)
= 2(2mn+m+n)+1
= 2 P +1
proved

Where P=2mn+m+n

Example:-Prove that square of even integer is even integer.
Solution:-

Let $p=2m^2$……………………………………………(1)
Where m is integer value.

Squaring both side of equation (1) we get

$$P^2=(2m^2)^2$$

$P^2=4\ m^4$

$P^2=2*2\ m^4$

$2m^2$ is an integer therefore $2M^4$ is also integer.

Hence 2*(Even), must be even integer.

Indirect Method:-

$p \Rightarrow q$

In this method we will assume that q is false and then show that p is false.

Example:-

Proove that if x, y∈Z, such that xy is odd, then both x and y are odd.

Proof:-

Let p: xy is odd

q:both x and y are odd.

So

~p:xy is even and

~q: x is even or y is even or both are even.

We will prove that by using contradiction technique

Let x=2n for some n∈N

Therefore xy=2ny

Therefore xy is even, By definition

That is, ~p is true

Mathematical Induction:-

A means of proving a theorem by showing that if it is true of any particular case it is true of the next case in a series, and then showing that it is indeed true in one particular case.

Example:-1

Prove that sum of natural numbers

For any positive integer n, $1 + 2 + ... + n = n(n+1)/2$.

Example:-2

Prove that sum of square of natural numbers

For all positive integers n, $1^2 + 2^2 + ... + n^2 = (n)*(n+1)*(2n+1)/6$.

is correct

Example:-3

Prove that sum of cube of natural numbers

For all positive integers n, $1^3+2^3+3^3+4^3\ldots+n^3= \{n*(n+1)/2\}^2$ is correct

Problem 4:-

Prove that for any positive integer number n , $n^3 + 2n$ is divisible by 3

Problem 5:

Prove that $3n > n^2$ for $n = 1$, $n = 2$ and use the mathematical induction to prove that $3n > n^2$ for n a positive integer greater than 2.

Solution:-1

$1+2+3+4\ldots+n=n*(n+1)/2$ is correct………………(1)

Proof:-

We check for (1) is true for n=1

$n*(n+1)/2$

$=1*(1+1)/2$

$=2/2$

$=1$ true for n=1

We check for (1) is true for n=2

$S_n = n*(n+1)/2$

$=2*(2+1)/2$

$=6/2$

$=3$ true for n=2

Now let (1) is true for n=k

$S_k= k*(k+1)/2$……………………………………………(2)

Now we will show that (1) is true for n=k+1

$S_{k+1}= (k+1)*(k+1+1)/2$

$S_{k+1}= (k+1)*(k+2)/2$

$S_{k+1}=k(k+1)/2+2(k+1)/2$

$S_{k+1}=k(k+1)/2+ (k+1)$…………………………………....(3)

From equation (2) , equation (3) become

$S_{k+1}= S_k+ (k+1)$…………………………………………...(4)

Equation (4) is true for n=(k+1)

Therefore equation (2) is for true n=k.

Hence, equation (1) is also true for all k=n.

Solution:2

Prove that sum of square of natural numbers

$1^2+2^2+3^2+4^2…+n^2=n/6 *(n+1)(2n+1)$ is correct **…………………(1)**

Proof:-

We check for (1) is true for n=1

$S_n =(n/6) *(n+1)(2n+1)$

$=(1/6)*(1+1)(2*1+1)$

$=(1/6)*6$

$=1$

We check for (1) is true for n=2

$S_n =(2/6) *(2+1)(2*2+1)$

$=(2/6)*(3)*5$

$=5$

Now let (1) is true for n=k

$S_k= (k/6) *(k+1)(2k+1)$……………………………………………(2)

Now we will show that (1) is true for n=k+1

S_{k+1}= {(k+1)/6}*(k+1+1){(2(k+1)+1})

S_{k+1}= {(k+1)/6}*(k+2){2k+2+1}

S_{k+1}= {(k+1)/6}*(k+2){2k+1+2}

S_{k+1}= {k(k+1)/6}+{2(k+1)/6}[{2k+1+2}]

S_{k+1}= {k(k+1)/6}+2(k+1)/6}[{(2k+1)+2}]

S_{k+1}= {k(k+1)/6}(2k+1)+ {2(k+1)/6}*(2k+1)+ {2*k(k+1)/6}+{2*2(k+1)/6}

S_{k+1}=[{k(k+1)/6}(2k+1)]+(k+1)/6{4(k+1)+2k+2}……………………………(3)

From equation (2) , equation (3) become

S_{k+1}= S_k +(k+1)/6[4k+4+2k+2]

S_{k+1}= S_k +(k+1)/6[6k+6]

S_{k+1}= S_k +(k+1)(k+1)

S_{k+1}= S_k +$(k+1)^2$

From equation (2) , equation (3) become

S_{k+1}= S_k +$(k+1)^2$……………………………………….....(4)

Equation (4) is true for n=(k+1)

Therefore equation (2) is also true k=n.

Hence, equation is true for all n.

Solution-3

Prove that sum of cube of natural numbers

$1^3+2^3+3^3+4^3…+n^3$= $\{n*(n+1)/2\}^2$ is correct**……………………………….(1)**

Proof:-

We check for (1) is true for n=1

S_n =$\{n*(n+1)/2\}^2$

S_1 =$\{1*(1+1)/2\}^2$

S_1 =1 true

We check for (1) is true for n=2

S_2 =$\{2*(2+1)/2\}^2$

S_2 =9 true

Now let (1) is true for n=k

S_k= $\{k*(k+1)/2\}^2$……………………………………………..(2)

Now we will show that (1) is true for n=k+1

S_{k+1}= $\{(k+1)*(k+1+1)/2\}^2$……………………………………(3)

S_{k+1}=$\{(k+1)*(k+2)/2\}^2$

S_{k+1}=$\{k(k+1)/2+2(k+1)/2\}^2$

S_{k+1}= $[k(k+1)/2]^2$+$[(k+1)^2]$+2*(k(k+1)/2(k+1)

From equation(2)

S_{k+1}= S_k + $(k+1)^2$+k$(k+1)^2$

$$S_{k+1} = S_k + (k+1)^2(1+k)$$
$$S_{k+1} = S_{k+1} + (k+1)^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Equation (4) is true for n=(k+1)

Therefore equation (2) is also true k=n.

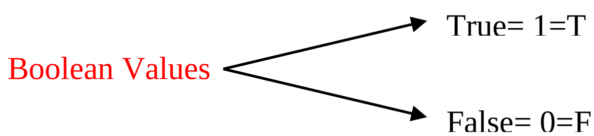
Hence, equation is true for all n.

Principle of Induction:-

Show that following formulas are true using induction method

- Sum of n natural numbers
  $S_n = 1+2+3+4+\ldots+n = n(n+1)/2$
- Sum of square of n natural numbers
  $S_n = 1^2+2^2+3^2+4^2+\ldots+n^2 = n/6(n+1)(2n+1)$
- Sum of cube of n natural numbers
  $S_n = 1^3+2^3+3^3+4^3+\ldots+n^3 = \{n(n+1)/2\}^2$

Boolean algebra and Circuit:-

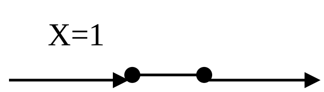


A Boolean algebra B is an algebraic structure which consists of a set X having two binary operations denoted by v and Λ,one unary operations denoted by ′ or ~ or ┐ and two specially defined elements **O** and **I.**Which satisfy the following five laws for all x,y,zЄX

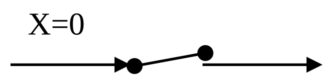
B1:-Associative Laws	:	$x \vee (y \vee z) = (x \vee y) \vee z$
	:	$x \wedge (y \wedge z) = (x \wedge y) \wedge z$
B2:-Commutative Laws	:	$x \vee y = y \vee x$
		$x \wedge y = y \wedge x$
B3:-Distributative Laws	:	$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$
	:	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
B4:-Identative Laws	:	$x \vee \mathbf{0} = \mathbf{x}$
	:	$x \wedge \mathbf{I} = \mathbf{x}$
B5:-Complementation Laws	:	$x \vee x' = \mathbf{O}$
	:	$x \wedge x' = \mathbf{I}$

Switching Circuit:-

State-1 Open State (1) /ON

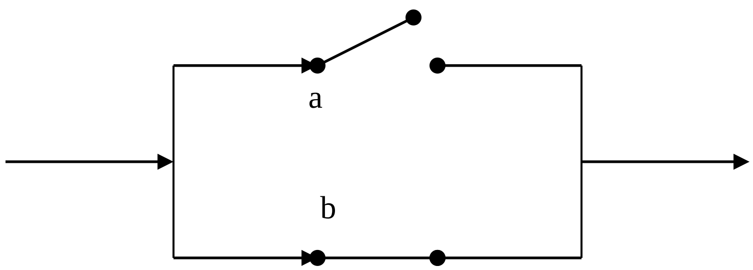


State-2 Closed State (0)/OFF

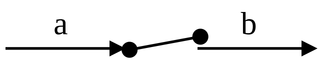


Circuit Connections:-

Parallel Connection



Series Connection:-



Theorem Based On Boolean algebra:-

Idempotent Laws	:	$x \vee x \vee x \vee x \vee x \vee x \vee x = x$
	:	$x \wedge x \wedge x \wedge x \wedge x \wedge x \wedge x \wedge x = x$
Absorption Laws	:	$x \vee (x \wedge y) = x$
	:	$x \wedge (x \vee y) = x$
Involution Laws	:	$(x')' = x$
De Morgan's Laws	:	$(x \vee y)' = x' \wedge y'$
	:	$(x \wedge y)' = x' \vee y'$

Dual of Proposition/Statement:-

It is denoted by p^d.This is obtained by replacing $\wedge$ by v and v by $\wedge$

Example:-

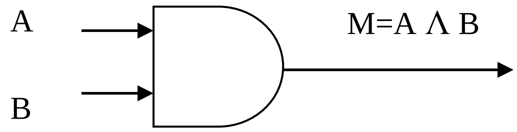
Dual of $(x \vee y) \wedge y$ is $(x \wedge y) \vee y$

Logic Gate Circuit:-

A logic gate is an elementary building block of a digital circuit. Most logic gates have two inputs and one output. At any given moment, every terminal is in one of the two binary conditions low (0) or high (1), represented by different voltage levels. There are following types of gates

- AND gate
- OR gate
- NOT gate
- NOR gate
- NAND gate
- EX-OR gate
- EX-NOR gate

AND GATE:-



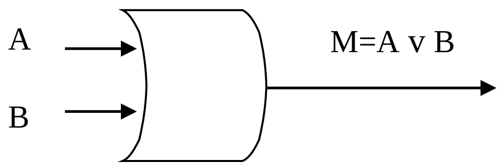
TRUTH TABLE:-

Let n=input variable
Input Signal=2^n

n=2(A, B) = 2^2=4(0, 1, 2, 3)

A	B	M=A Λ B
F	F	F
F	T	F
T	F	F
T	T	T

OR GATE:-

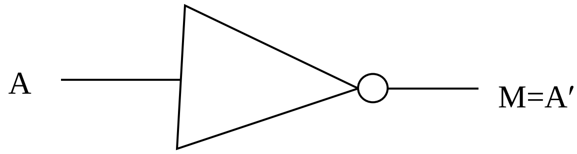


TRUTH TABLE:-

Let n=input variable
Input variable=2^n
n=2(A, B) = 2^2=4(0, 1, 2, 3)

A	B	M=A v B
F	F	F
F	T	T
T	F	T
T	T	T

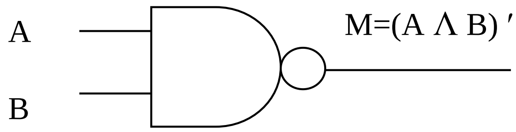
NOT GATE:-



TRUTH TABLE:-
Let n=input variable
Input variable=2^n
n=1(A)= 2^1=2(0,1)

A	M=A′
F	T
T	F

NAND GATE:-



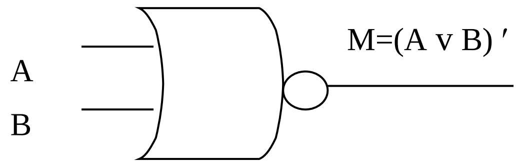
TRUTH TABLE:-

Let n=input variable
Input variable=2n

n=2(A, B) = 2^2=4(0, 1, 2, 3)

A	B	M1=A Λ B	M=(A Λ B) ′
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	F

NOR GATE:-

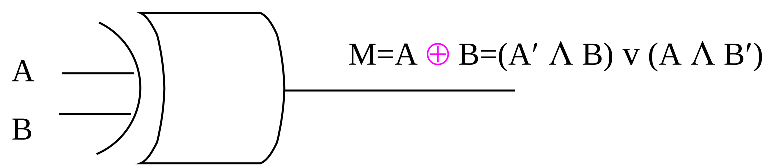


TRUTH TABLE:-

Let n=input variable
Input variable=2n
n=2(A, B) = 2^2=4(0, 1, 2, 3)

A	B	F1=A v B	F=(A v B) ′
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

EX-OR GATE:-



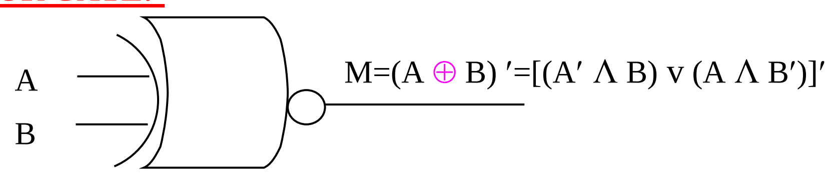
TRUTH TABLE:-

Let n=input variable
Input variable=2^n

n=2(A, B) = 2^2=4(0, 1, 2, 3)

A	B	A′	B′	A′Λ B	A Λ B′	(A′ Λ B) v (A Λ B′)
F	F	T	T	F	F	F
F	T	T	F	T	F	T
T	F	F	T	F	T	T
T	T	F	F	F	F	F

EX-NOR GATE:-



TRUTH TABLE:-

Let n=input variable

Input variable=2^n

n=2(A, B) = 2^2=4(0, 1, 2, 3)

A	B	A′	B′	A′ ∧ B	A ∧ B′	A ⊕ B	M= (A ⊕ B) ′
F	F	T	T	F	F	F	T
F	T	T	F	T	F	T	F
T	F	F	T	F	T	T	F
T	T	F	F	F	F	F	T

Boolean Function:-

This provides information about the overall functioning of the corresponding logic circuit. More generally, each Boolean expression(x1, x2, x3, x4, x5,…x_k) in k variables, where each variable can take values from the two-element Boolean algebra β,defines a function

F:B^k→ B:f(e1, e2, e3,e4,e5,e6…e_k)

Any such function is called a Boolean function.

Example:-1

Let f:β^2→ β denote the function which is defined by the Boolean expression

X(x1,x2)= x1′ Λ x2′

Write the value of f in tabular form

Solution:-

X(x1,x2)= x1` Λ x2`

N=2

2^2=4(0, 1, 2, 3)

X (0,0)= 0′ Λ 0′= 1 Λ 1=1

X (0,1)= 0′ Λ 1′= 1 Λ 0=0

X (1,0)= 1′ Λ 0′= 0 Λ 1=0

X (1,1)= 1′ Λ 1′= 0 Λ 0=0

Example:-2 Find all the values of Boolean function g:β→ β defined by the Boolean expression

(x1 Λ x2) v (x1 Λ x3′)

N=3

2^3=8(0, 1, 2, 3, 4,5, 6, 7)

g(0,0,0)= (0 Λ 0) v (0 Λ 0′)=(0 Λ 0) v (0 Λ 1)=0v0=0

g(0,0,1)= (0 Λ 0) v (0 Λ 1′)=(0 Λ 0) v (0 Λ 0)= 0v0=0

g(0,1,0)= (0 Λ 1) v (0 Λ 0′)=(0 Λ 0) v (0 Λ 1)= 0v0=0

g(0,1,1)= (0 Λ 1) v (0 Λ 1′)=(0 Λ 0) v (0 Λ 0)= 0v0=0

g(1,0,0)= (1 Λ 0) v (1 Λ 0′)=(1 Λ 0) v (1 Λ 1)= 0v1=1

g(1,0,1)= (1 Λ 0) v (1 Λ 1′)=(1 Λ 0) v (1 Λ 0)= 1v0=0

g(1,1,0)= (1 Λ 1) v (1 Λ 0′)=(1 Λ 1) v (1 Λ 1)= 1v1=1

g(1,1,1)= (1 Λ 1) v (1 Λ 1′)=(1 Λ 1) v (1 Λ 0)= 1v0=1

Introduction of Sets:- Most Important

It is a well defined collection. The objects belonging to a set are called elements or members of that set.

Example:-1 Set of state of India:-

S= {UP, Bihar, MP, Maharashtra, Andhra Pradesh, Keral, Rajasthan}.

Example:-2 Set of district of UP:-
C= {Varanasi,Allahabad,Chandauli,Mirzapur,**B**areli,Baliya,Gorakhpur }.

Example:-3 Set of stationary:-
A= {pencil,eraser, paper,pen,sharpner}.

Technique of representing set:-

- Tabular form
- Set-builder form

Example of tabular form:-

Example:-1 Set of seven state of India:-
S= {UP, Bihar, MP, Maharashtra, Andhra Pradesh, Keral, Rajasthan}.
Example:-2 Set of ten Fibonacci numbers:-
F= {0,1,1,2,3,5,8,13,21,34 }.
Example:-3 Set of ten Prime numbers:-
P= {2,3,5,7,11,13,17,23,29,31,37,41,43 }.
Example:-4 Set of twenty natural numbers:-
P= {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20 }.

Set-builder form:-

Such type of set looks like {x:p(x)} or {x|p(x)}
Where p(x) is the property for x.

Example:-1 Set of all natural numbers:-
N={n : n=1,2,3,4,…}
Example:-2 Set of all female in india:-
F={f : female f belong in India only}
Example:-3 Set of all male in india:-
F={m : male m belong in India only}
Example:-4 Set of all odd numbers:-
ODD={n : n=2n+1,where n=0,1,2,3,4,5,6,7,8…}
Example:-5 Set of all even numbers:-
EVEN={n :n=2n,where n=1,2,3,4,5,6,7,8…}

Types of Set:-

- **Empty set/Null set/void set:-**

Such types of set no contain any element.It is represented by Ø or { }
Example:-
A={x: | x is an integer between 1 to 10 which is divisible by 13}.
A= { }
Or
A= Ø

- **Finite set:-**

A set having a finite number of elements is called a finite set.

Example:-1
S= {s | s represent state of India}.
Example:-2
D= {d | d represent district of India}.
Example:-3
Student= {st | st represent student of UPRTOU}.

- **Infinite set:-**

A set having a infinitely many elements is called an infinite set.

Example:-

Set of natural numbers
Set of prime numbers
Set of odd numbers
Set of rice in India
Set of Fibonacci numbers
Set of stars in galaxy
Set of ants in India

Subset and Super set:-

A set is called to be a subset of set B if each element of set A is also an element of set B. It is denoted by

Example:-1

A= {3, 5, 6,7, 8, 9, 12}
B={3,6,7}
Set B is the subset of set A. And A is the super set of set b.
ie $B \subseteq A$

Equal Set:-

Let A={2,3, 4,5,6}
B={2,3,4,5,6}
A=B
B=A

Proper Set:-

A set B is said to be a proper subset of set B if B is the subset of set A. And B and A are not equal.It is denoted by if $A \subset B$

We say that A is a proper subset of B and we write $A \subset B$ strictly
if $A \subset B$
(There exists at least one element $b \in B$ such that $b \notin A$.)
Example:-1

A= {2, 3, 4, 5, 6, 8, 9, 88}
B= {2, 3, 4, 5, 6}
B is the proper subset of Set A.

Number of subset of given Set:-

If n is the number of elements of given set.Then its total subset will be 2^n
Example:-

A={2,4,5}
Number of subset=2^3=8
A1={2}
A2={4}
A3={5}
A4={2,4}
A5={4,5}
A6={2,5}
A7={2,4,5}
A8={}

Powerset:-

The power set of A is the set of all the subsets of A,Then it is denoted by P(A).

Example:-1

A={2,4,5}

P(A)={ {2},{4},{5},{2,4},{4,5},{2,5},{2,4,5},{}}

Example:-2

A={3,7,9,11}

A1={3},A2={7},A3={9},A4={11},A5={3,7},A6={7,9},A7={9,11},A8={3,11},A9={3,9},
A10={7,11},A11={3,7,9},A12={7,9,11},A13={3,7,11},A14={3,9,11},A15={3,7,9,11},
A16={}

P(A)={{3},{7},{9},{11},{3,7},{7,9},{9,11},{3,11},{3,9},{7,11},{3,7,9},{7,9,11},
{3,7,11},{3,9,11},{3,7,9,11},{}}

Example:-3

P(∅) = { ∅ }. (P(∅) is not empty, it has exactly one element, the ∅ .)

Operation On Set-

- Union of sets
- Intersection of sets
- Difference of two sets
- Complement of a set
- Symmetric difference of two sets

Union of set:-

If A and B are two sets.Its union defined by

AUB= {x| x ∈ A or x ∈ B }

Example:-

A= {2, 3, 4, 5, 6, 8, 9, 88}

B= {2, 3, 4, 5, 6,13}

AUB={2,3,4,5,6,8,9,13,88}

Intersection of set:-

If A and B are two sets.Its intersection defined by

A∩B= {x| x ∈ A and x ∈ B }

Example:-

A= {2, 3, 4, 5, 6, 8, 9, 88}

B= {2, 3, 4, 5, 6, 13}

A∩B= {2, 3, 4, 5, 6}

Difference of two sets:-

If A and B are two sets.Its difference defined by

A-B= A\B = {x| x is the element of set A and x is not the element of set B}

B-A= B\A ={x| x is not the element of set A and x is the element of set B}

Example:-

A= {2, 3, 4, 5, 6, 8, 9, 88}

B= {2, 3, 4, 5, 6,13}

A-B={8,9,88}

B-A={13}

Complement of set:-

Let U is universal set and A is the set which contain some element of universal set. Its compliment denoted by A` or A^c.

A`=A^c.=U-A

Example:-

U= {1, 2, 3, 4, 5, 6, 7, 8, 9}
A= {3, 5,7}
U-A=A^c= {1, 2, 4, 6, 8, 9}

Symmetric Difference:-

Let A and B are two sets.Its symmetric difference defined as

AΔB= (A-B) ∪ (B-A)= (A/B) ∪ (B/A)

Example:

A = {3, 5, 7}
B = {3, 7, 8, 9}
A-B = A/B = {5}
B-A = (B/A)= {8, 9}

AΔB = (A-B) ∪ (B-A)
= {5}U{8,9}
= {5, 8,9}

Properties of sets:-

For any universal set U and subsets A,B and C of U,The following properties hold.

- **Associative properties:-**

AU(BUC)=(AUB)UC
A∩(B∩C) = (A∩B) ∩C

Example:-1

A = {3, 5,6, 7,11}
B = {3, 7, 8, 9}
C = {4, 7, 8, 19}

AU(BUC)=(AUB)UC

LHS:-
AU(BUC)
BUC={3,4,7,8,9,19}
AU(BUC)={3,4,5,6,7,8,9,11,19}

RHS:-
(AUB)UC
AUB={3,5,6,7,8,9,11}
(AUB)UC={3,4,5,6,7,8,9,11,19}
LHS=RHS
Example:-2
A∩(B∩C) = (A∩B) ∩C
LHS:-
A∩(B∩C)
B∩C={7,8}
A∩(B∩C)={7}
RHS:-
(A∩B) ∩C
A∩B={ 3,7}
(A∩B) ∩C={7}

- **Commutative properties:-**

AUB= BUA
A∩B= B∩A

- **Identity:-**

  AU ∅=A

  A∩ U=A

Example:-

  U= {1, 2, 3, 4, 5, 6, 7, 8, 9}

  A= {1, 2, 3}

  A∩ U= {1, 2, 3} =A

- **Complement:-**

  AUA^c=U

  A∩**A^c=** ∅

Example:-

  U= {1, 2, 3, 4, 5, 6, 7, 8, 9}

  A= {1, 2, 3}

  A^c=U-A={4,5,6,7,8,9}

  AUA^c={1, 2, 3, 4, 5, 6, 7, 8, 9}= U

  A∩**A^c=** ∅={ }

- **Distributative Properties:-**

  AU(B ∩C)=(AUB) ∩(AUC)

  A∩(B UC)=(A∩B) U(A∩C)

Example:-1

  A = {3, 5,6, 7,11}

  B = {3, 7, 8, 9}

  C = {4, 7, 8, 19}

AU(B ∩C)=(AUB) ∩(AUC)

LHS

  AU(B ∩C)

  B ∩C= {7, 8}

  AU (B ∩C) = {3, 5, 6, 7, 8, 11}

RHS

  (AUB) ∩(AUC)

  AUB=**{**3,5 ,6,7,8,9,11}

  AUC= {3, 4, 5, 6, 7, 8, 11,19}

  (AUB) ∩(AUC)={3, 5,6,7,8,11}

Example:-2

  A = {3, 5,6, 7,11}

  B = {3, 7, 8, 9}

  C = {4, 7, 8, 19}

  A∩(B UC)=(A∩B) U(A∩C)

LHS

  A∩(B UC)

  B UC={3,4,7,8,9,19}

  A∩(B UC)={3,7}

RHS

  (A∩B) U(A∩C)

  A∩B={3,7}

  A∩C={7}

  (A∩B) U(A∩C)={3,7}

De Morgan's law:-First Form

  $(AUB)^c$=A^c∩B^c

  $(A∩B)^c$=A^cUB^c

De Morgan's law:-Second Form

A-(BUC)=(A-B) ∩(A-C)
A-(B∩C)=(A-B) U (A-C)

Example:-

A = {3, 5,6, 7}
B = {3, 7, 8, 9}
C = {4, 7, 8}

A-(BUC)=(A-B) ∩(A-C)

LHS

A-(BUC)
BUC={3,4,7,8,9}
A-(BUC)={5,6}

RHS

(A-B) ∩(A-C)
A-B={5,6}
A-C={3,5,6}
(A-B) ∩(A-C)={5,6}

LHS=RHS

Dual Of Sets:-

U→∩
∩→ U

Example:-1

Find the dual of A∩(B∩C)

Solution:-

Dual of A∩(B∩C) is AU(BUC)

Example:-2

Find the dual of AU(BUC)
Dual of AU(BUC) is A∩(B∩C)

Relations:-

Cartesian product:-

An ordered pair, usually denoted by (x,y) is a pair of elements x and y of some sets.This is ordered in sense of that(x,y)≠(y,x).

Example:-

A={1,2,3}
B={p,q,r}

A×B={(1,p),(1,q),(1,r),(2,p),(2,q),(2,r),(3,p),(3,q),(3,r)}

B×A={(p,1),(p,2),(p,3),(q,1),(q,2),(q,3),(r,1),(r,2),(r,3)}

A×B≠ B×A

Note:-

(1,p)=^1R_p

Example:-

If X={a,b,c} & Y= {1,2,3}
Find

- X×**X={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)}**
- X×**Y={(a,1),(a,2),(a,3),(b,1),(b,2),(b,3),(c,1),(c,2),(c,3)}**
- Y×**X={(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)}**
- Y×**Y={(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}**

Definition of relation :- Most Important

A relation between two sets A and B is subset of A×B.Any subset of A×A is a relation on the set A.

Example:-
A={1,2,3}
B={p,q}

A×B={(1,p),(1,q),(2,p),(2,q),(3,p),(3,q)}
Here (1,q),(2,p),(2,q),(3,p) is relation on A×B

Theorem Based On Relation:-
The total number of relations between a finite set A and a finite set B is 2^{mn}
Example:-
Let A= {3, 4, 5, 6, 7}
B={p,q,r,s,t}
Total number of relation=$2^{mn}=2^{5*5}=2^{25}$

Properties of relations

1:- Reflexive relations

A relation R on set A is called a reflexive relation if (a, a) ЄR for aЄA.

2:- Symmetric relations

A relation R on set A is called a symmetric relation if (a,b) ЄR=>(b,a) ЄR

3:- Transitive relations

A relation R on set A is called a transitive relation if whenever (a,b) ЄR and (b,c) ЄR then (a,c) ЄR for all a,b,cЄA

Equivalence Relations:-

A relation R on a set A is called an equivalence relation if and only if

- R is reflexive ie, for all a ЄR,(a,a) ЄR
- R is symmetric,ie.,(a,b) ЄR=>(b,a) ЄR, for all a,b ЄA and
- R is transitive,ie.(a,b) ЄR and (b,c) ЄR=>(a,c) ЄR for all a,b,cЄA

Functions/Mapping:- Most Important

It is a special kind of relation.A function from a non-empty set A to a non-empty set B is a subset R of A×**B such that for each a** ЄA there exist a unique bЄB such that (a,b) ЄR.This relation must satisfies the following two conditions:

- For each a ЄA,There is some bЄB such that (a,b) ЄR
- If(a,b) ЄR and (a,b') ЄR then b=b'

Or

Let A and b be non-empty sets.A function f from A to B is a rule that assigns to each element x in exactly one element y in B.We write this as f:A→B.

Example:-

Let A= {1, 2, 3, 4} Domain
B= {1, 4, 9, 16} Co-domain/Range

$f(x)=x^2$
f:A→B.
$f(1)=1^2=1$
$f(2)=2^2=4$
$f(3)=3^2=9$

f(4)=4^2=16

Example of Functions:-

- **Identity function**

f(x)=x is called identity function
x=1, 2,3,4,5,6,7,8,9…

f(1)=1
f(2)=2
f(3)=3
f(4)=4
f(5)=5
.
.
.

- **Constant function**

f(x)=c1 is called constant function

x=1, 2, 3,4,5,6,7,8,9…
f(1)=c1
f(2)=c1
f(3)=c1
f(4)=c1
f(5)=c1
.
.
.

- **Greatest function /floor function**

It is denoted as f(x)=[x],where [x] is the greatest integer<=x

Concept of Real Line (R):-
-∞……………………..0………………………+∞

Example:-

f(x)=[x]
f(4.3)=[4.3]=4
f(2.1)=[2.1]=2
f(-3.4)=[-3.4]=-4
f(-8.4)=[-8.4]=-9

- **Absolute function/Modulus Function :-**

It is denoted by f: R→R:f(x)=|x|
It is defined as
f(x)=x if x>=0
f(x)=-x if x<0

Example:-
f(-6)=6
f(7)=7

Concept of Real numbers:-

Whole numbers between -∞ to +∞ are called real numbers.

-∞ ←——— 0 ———→ +∞

Real number represented by symbol R.

How to Calculate domain and co-domain/Range:-

Example:-1

f(x)=1/(1-x)
put x=1
f(1)=1/(1-1)= 1/0=∞ (undefined)
Domain →whole real numbers. That is, Domain=R=[-∞,+∞]
Co-domain=R-{1}=[-∞,+∞]-{1}

Example:-2

f(x)=x^2 +1
Domain→whole real numbers. That is, Domain=R=[-∞,+∞]
Co-domain=R=[-∞,+∞]

Types of function:-

- Onto Mapping/Function

  A mapping f:A→B is said to an onto (or surjective) mapping if f(A)=B.That is range and co-domain coincide.

  Example:-

  f(x)=x+1 Its relation f:Z →Z (Z indicate integer values)

- Injective Function

  A mapping f:A→B is said to be injective (or one –one) if the images of distinct elements of A under f are distinct.If x1≠x2, then f(x1) ≠f(x2)

  Example:-

  f(x)=x^2 +1

- Bijective Function

  A mapping f:A→B is said to be bijective (or one-one onto).

  Example:-

  f(x)=x +1

Permutation:-

A mapping f: A→B is said to be permutation on the set A.
Let A= {a1, a2, a3,…} and f is a bijection from A onto A that maps a_i to $f(a_i)$ Then we write f as

$$f=\begin{pmatrix} a1 & a2 & a3 & a4 & \dots \\ f(a1) & f(a2) & f(a3) & f(a4) & \dots \end{pmatrix}$$

Identity Mapping

$$f=\begin{pmatrix} a1 & a2 & a3 & a4 & \dots \\ a1 & a2 & a3 & a4 & \dots \end{pmatrix}$$

Inverse Function:-

If f: A→B is a function then its inverse is defined as
f^{-1}: B→A

$$f=\begin{pmatrix} a1 & a2 & a3 & a4 & \dots \\ b1 & b2 & b3 & b4 & \dots \end{pmatrix}$$

$$f^{-1}=\begin{pmatrix} b1 & b2 & b3 & b4 & \dots \\ a1 & a2 & a3 & a4 & \dots \end{pmatrix}$$

Example:-1

$$f=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$$g=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$fg=\begin{pmatrix} 1 & 2 & 3 \\ \mathbf{1} & \mathbf{3} & \mathbf{2} \end{pmatrix}$$

Example:-2

$$gf=\begin{pmatrix} 1 & 2 & 3 \\ \mathbf{3} & \mathbf{2} & \mathbf{1} \end{pmatrix}$$

Example:-3

$$f*f=\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Example:-4

$$f=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$

$$g=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$h=\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 2 & 1 & 3\end{pmatrix}$$

Find out f(gh),(fg)h

$$gh=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 3 & 4\end{pmatrix}$$

$$f(gh)=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{pmatrix}$$

Example:-5 Find out inverse of f^{-1}, g^{-1}, $f(gh)^{-1}$

$$f^{-1}=\begin{pmatrix}1 & 3 & 4 & 2\\ 1 & 2 & 3 & 4\end{pmatrix}$$

$$g^{-1}=\begin{pmatrix}2 & 3 & 4 & 1\\ 1 & 2 & 3 & 4\end{pmatrix}$$

$$f(gh)^{-1}=\begin{pmatrix}1 & 2 & 3 & 4\\ \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{1}\end{pmatrix}$$

Operation on functions:-

Let f(x) and g(x) are two functions then following types of operations must be exist.

- (fg)(x)=f(x) g(x)
- (f+g)(x)=f(x)+ g(x)
- (f-g)(x)=f(x)- g(x)
- (f/g)(x)=f(x)/ g(x) where g(x) ≠0

Example:-1

If f(x)=sinx +cosx
g(x)=tan(x)

f(x)+g(x)= sinx +cosx+ tan(x)
f(x)*g(x)=(sinx +cosx) tan(x)
f(x)/g(x)=(sinx +cosx)/ tan(x)

Composition functions:-

Let f(x) and g(x) are two functions then its composite function defined as

$$fog(x)=f(g(x))$$
$$gof(x)=g(f(x))$$

Example:-1

If $f(x)=x^2+x+3$
$g(x)=\tan(x)$

$fog=f(g(x))$
$=f(\tan(x))$
$=(\tan(x))^2+\tan(x)+3$

$gof=g(f(x))$
$=g(x^2+x+3)$
$=\tan(x^2+x+3)$

Product of two permutations:-

Example1

$$f=\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}$$

$$g=\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix}$$

$$fg=\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix}$$

$$gf=\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}$$

$$f^2=ff=\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$$

Example2

$$f=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$

$$g=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$h=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$$

$$fg=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$$

$$(fg)h=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$